APC8 Quantum Field Theory - Take Home Final

(Problem 1) Consider the following classical Lagrangian for a massive vector particle

$$\mathcal{L} = -\frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} + \frac{1}{2} M^2 A_\alpha A^\alpha - A_\alpha J^\alpha \ ,$$

where $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$, A_α is the vector potential and J^α is an external source.

(a) (10 points) Show that A_ν satisfies the so-called Proca equation

$$\left\{ g^{\mu\nu} \left(\Box + M^2 \right) - \partial^\mu \partial^\nu \right\} A_\nu = J^\mu \ .$$

Assuming $\partial_\mu J^\mu = 0$, find the constraint equation imposed on A_ν.

(b) (10 points) For a static point source J_μ at the origin, show that the solution for the potential A_μ is

$$A_0(r) = \frac{-ie}{4\pi^2 r} \int_{-\infty}^{\infty} \frac{k dk}{k^2 + M^2} e^{ikr} \ , \quad A_i = 0 \ .$$

(b) (10 points) Evaluate the above integral using contour integration to obtain an explicit form of $A_0(r)$. Show that one can reproduce the Coulomb potential by taking $M \to 0$.

(d) (10 points) The propagator $\Pi^{\mu\nu} = (g^{\mu\nu} (\Box + M^2) - \partial^\mu \partial^\nu)^{-1}$ is defined as

$$\left(g^{\mu\alpha} \left(\Box_x + M^2 \right) - \partial_x^\mu \partial_x^\alpha \right) \Pi_{\alpha\nu}(x, y) = g_\nu^\mu \delta^4(x - y) \ .$$

Show that the solution is

$$\Pi_{\alpha\nu}(x, y) = \int \frac{d^4 k}{(2\pi)^4} \frac{-1}{k^2 - M^2} \left(g_{\alpha\nu} - \frac{k_\alpha k_\nu}{M^2} \right) e^{ik\cdot(x-y)} \ .$$

Can you guess what the Feynman propagator is?

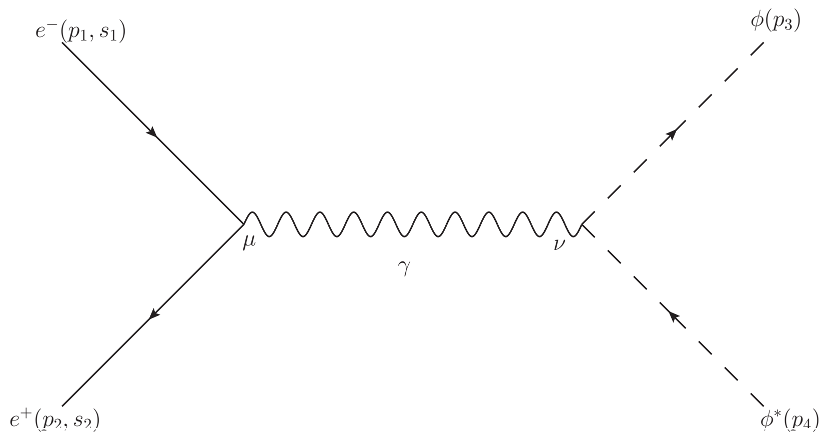
(Problem 2) (20 points) Consider the following interaction Lagrangian for a real quantum scalar field ϕ

$$\mathcal{L}_{\text{int}} = -\frac{1}{4!} \lambda \, \phi^4 \ .$$

Use the Lagrangian approach to compute the following 2-point function

$$\langle \Omega | T\{ \phi(x_1) \phi(x_2) \} | \Omega \rangle$$

to *first* order in the coupling λ.



(Problem 3) Consider the following process

$$e^-(p_1, s_1) + e^+(p_2, s_2) \to \phi(p_3) + \phi^*(p_4)$$

in quantum electrodynamics to 2^{nd} order in perturbation theory, as shown above. Here e^- is the electron and ϕ is a hypothetical scalar particle with the same charge as the electron.
(a) (10 points) Write down the Lorentz invariant matrix element $i\mathcal{M}$ for this process using the Feynman rules listed in the text.
(b) (10 points) Compute the unpolarized matrix element squared

$$\frac{1}{4} \sum_{s_1, s_2} |\mathcal{M}|^2 \, .$$

Express your answer in terms of the Mandelstam variables
$s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_1 - p_4)^2$.
(c) (10 points) Calculate the unpolarized differential cross section $d\sigma/d\Omega$ for the process in the center-of-mass frame.
(d) (10 points) Set the masses $m_e = 0$ and $m_\phi = 0$ and obtain the total cross section in the center-of-mass frame.

Due 9:00 AM, May 3, 2017

Be Honest & Good Luck!